(1) Find the inverse

(a) $y = \sqrt[3]{x+5}$

$x = \sqrt[3]{y+5}$

$x^3 = y+5$

$\boxed{x^3 - 5 = y}$

(b) $y = \arctan\left(\frac{x}{3}\right)$

tan tan

$x = \arctan\left(\frac{y}{3}\right)$

$\tan x = \frac{y}{3}$

$\boxed{3\tan x = y}$

(2) Find the domain/range and evaluate at $x=1$

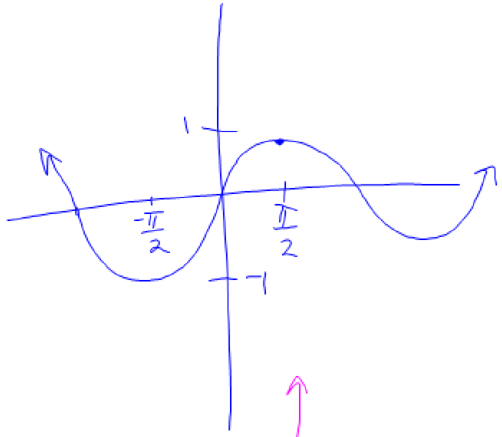
(a) $y = \sin^{-1}(x)$

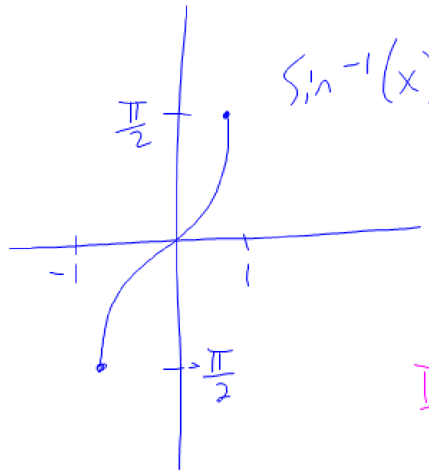
(b) $y = \arccos(x)$

(c) $y = \tan^{-1}(x)$

(d) $y = \sec^{-1}(x)$

(e) $y = \tan(\arctan(x))$

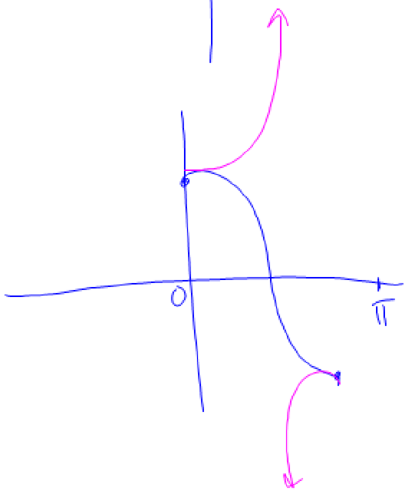


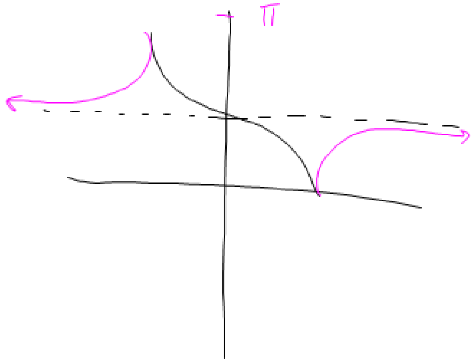


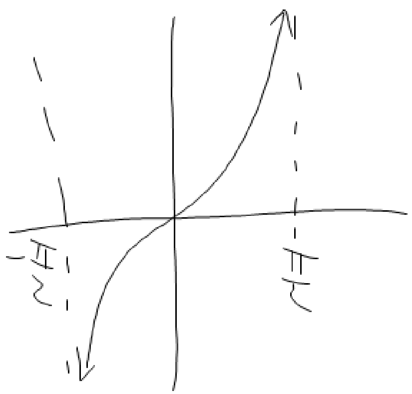
$\sin^{-1}(x)$ D: $[-1, 1]$

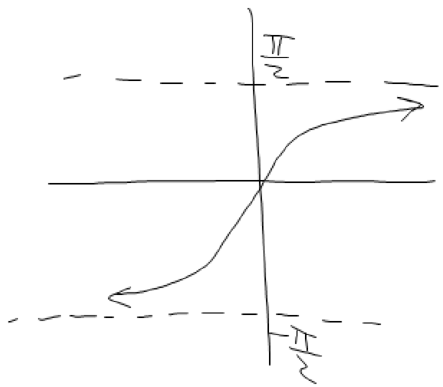
R: $[-\frac{\pi}{2}, \frac{\pi}{2}]$

D: $(-\infty, -1] \cup [1, \infty)$









(23) $y = \sec^{-1}(x)$ at $x = 2$

$$y' = \frac{1}{|x|\sqrt{x^2-1}} \text{ at } x = 2 \Rightarrow \frac{1}{2\sqrt{2^2-1}}$$

tangent $= \frac{\sqrt{3}}{6}(x-2) + \frac{\pi}{3}$

$$L(x) = f'(a)(x-a) + f(a) \Rightarrow \frac{1}{2\sqrt{3}} \Rightarrow \frac{\sqrt{3}}{6} \text{ slope}$$

orig point $\left(2, \frac{\pi}{3}\right)$

$$\sec^{-1}(x) = \cos^{-1}\left(\frac{1}{x}\right)$$

$$y = \sec^{-1}(2)$$

$$\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

5

$y = \sin^{-1}\left(\frac{3}{t^2}\right)$

$\frac{d}{dx} = \frac{3}{t^2} = \frac{d}{dx} 3t^{-2} = -2 \cdot 3 \cdot t^{-3}$

$$\frac{dy}{dt} = \frac{1}{\sqrt{1-\left(\frac{3}{t^2}\right)^2}} \cdot \frac{3 \cdot -2}{t^3} = \frac{-6}{t^3\sqrt{1-\frac{9}{t^4}}}$$

$$\frac{-6}{t^3\sqrt{\frac{t^4}{t^4}-\frac{9}{t^4}}} = \frac{-6}{t^3\sqrt{\frac{t^4-9}{t^4}}} = \frac{-6}{t^{\cancel{3}} \cdot \frac{\sqrt{t^4-9}}{\cancel{t^2}}} = \frac{-6}{t\sqrt{t^4-9}}$$

(17) $y = \sec^{-1}\left(\frac{1}{t}\right)$

$$\frac{dy}{dx} = \frac{1}{\left|\frac{1}{t}\right|\sqrt{\left(\frac{1}{t}\right)^2 - 1}} \cdot -\frac{1}{t^2} = \frac{-1}{t^2\left|\frac{1}{t}\right|\sqrt{\frac{1}{t^2} - 1}}$$

$$= \frac{-1}{t^2\left|\frac{1}{t}\right|\sqrt{\frac{1}{t^2} - \frac{t^2}{t^2}}} = \frac{-1}{t^2\left|\frac{1}{t}\right|\sqrt{\frac{1-t^2}{t^2}}} = \frac{-1}{t^{\cancel{2}}\left|\frac{1}{t}\right| \frac{\sqrt{1-t^2}}{\cancel{t}}}$$

$$\frac{-1}{t\left|\frac{1}{t}\right|\sqrt{1-t^2}} = \frac{-1}{\left|\frac{t}{t}\right|\sqrt{1-t^2}} = \frac{-1}{\sqrt{1-t^2}}, \quad t > 0$$

x	e^x	$\frac{d}{dx} e^x$
1	2.718	2.718
2	7.389	7.389
3	20.086	20.086
4		
5		

$Y_1 = 3^x$

$Y_2 = NDER(3^x, x, x)$

$1\left(1+\frac{1}{x}\right)^{\frac{1}{x}}$

$$\boxed{\frac{d}{dx} a^x = \ln(a)\, a^x}$$

3^{2x+1}

$\ln 3 \cdot 3^{2x+1} \cdot 2$

$$\boxed{\frac{d}{dx} a^u = \ln(a)\, a^u \cdot \frac{du}{dx}}$$

- derivatives were exponential and rate of growth is the base

- $\dfrac{\text{Derivative}}{\text{over } f(x)} = \text{constant} = \underline{\underline{\ln(\text{base})}}$

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$$\frac{d}{dx}e^x = e^x$$

Simplify

$3\ln x - \ln 3x + \ln(12x^2)$

$\ln x^3 - \ln 3x + \ln 12x^2 \longrightarrow \ln\left(\frac{x^3 \cdot 12x^2}{3x}\right) = \ln(4x^4)$

$= 4\ln(4x)$

Solve

ⓐ $3^x = 19$

$x = \log_3 19$

$\ln 3^x = \ln 19$

$x\ln 3 = \ln 19$

$x = \frac{\ln 19}{\ln 3}$

ⓑ $5^t \ln 5 = 18$

$5^t = \frac{18}{\ln 5}$

$\ln 5^t = \ln \frac{18}{\ln 5}$

$t = \frac{\ln \frac{18}{\ln 5}}{\ln 5}$

ⓒ $3^{x+1} = 2^x$

$\ln 3^{x+1} = \ln 2^x$

$(x+1)\ln 3 = x\ln 2$

$\ln 3x + \ln(3) = x\ln 2$

$\ln(3)x - \ln 2(x) + \ln 3$

$x(\ln(3) - \ln(2)) + \ln 3$

$x = \frac{-\ln 3}{\ln(3) - \ln(2)}$

≈ -2.710

① $\log x + \log y = \log(xy)$

② $\log x - \log y = \log\left(\frac{x}{y}\right)$

③ $2\log x = \log x^2$

Homework

- ☆ Read and understand 3.9 - bring ?'s
- If you need to, do some (more) 3.8
- Do it → Q.R. 3.9